You wish to find the root of the equation $x^3 = 2 - \ln x$ using Newton's Method. **SCORE: _____ / 7 PTS**

[a] Using the Intermediate Value Theorem, between which two **consecutive integers** must the root lie ?

$[1, 2]$

[b] If your first approximation of the root is x_1, what is the expression for the second approximation x_2 in terms of x_1 ?
Do NOT use f or f' notation.

$$X_2 = X_1 - \frac{x^3 + \ln x - 2}{3x^2 + \frac{1}{x}}$$

[c] Let x_1 be the lesser integer from [a]. Write down the sequence of approximations for the root that Newton's method generates.
Do NOT round off your answers.

$X_2 = 1.25$

$X_3 = 1.21787868$

$X_4 = 1.217214092$

$X_5 = 1.21721381 4 = X_6 = X_7 \ldots$

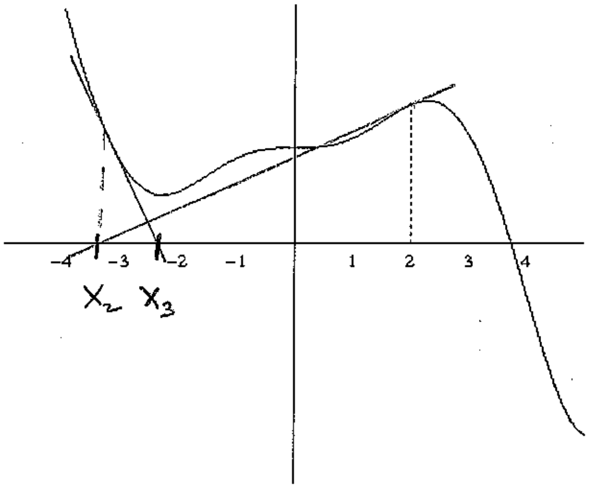
Eliminate the parameter to find a Cartesian equation of the parametric curve $\begin{array}{l} x = \cos 2t \\ y = \sin t \end{array}$. **SCORE: _____ / 4 PTS**

Your final answer must NOT use trigonometric nor inverse trigonometric functions.

$x = 1 - 2\sin^2 t$

$x = 1 - 2y^2$

You wish to use Newton's method on the graph of f below, with the first approximation $x_1 = 2$, to find the root **SCORE: _____ / 4 PTS** of the equation $f(x) = 0$. Find the approximations x_2 and x_3 **graphically**. (This shows why a good first approximation is important.)



Consider the parametric curve $\begin{aligned} x &= 6t^2 - t - 1 \\ y &= 2t^2 - t \end{aligned}$. **SCORE: _____ / 15 PTS**

You may use your calculator to check your answers, but your answers must be justified using calculus and algebra.

[a] Find the equation of the tangent line to the curve at the point $(6, 3)$.

$x = 6t^2 - t - 1 = 6$

$6t^2 - t - 7 = 0$

$(6t-7)(t+1) = 0$

$t = \frac{7}{6}, -1$

$y \neq 1 \qquad y = 3$

$\frac{dy}{dx} = \frac{4t-1}{12t-1}$

$\left.\frac{dy}{dx}\right|_{t=-1} = \frac{-5}{-13} = \frac{5}{13}$

$y - 3 = \frac{5}{13}(x-6)$

[b] Find $\frac{d^2y}{dx^2}$ at the point $(6, 3)$, and determine if the graph is concave up or down at that point.

$$\frac{\frac{d}{dt}\frac{4t-1}{12t-1}}{12t-1} = \frac{\frac{4(12t-1)-12(4t-1)}{(12t-1)^2}}{12t-1} = \frac{8}{(12t-1)^3}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=-1} = \frac{8}{(-13)^3} = -\frac{8}{13^3} < 0 \longrightarrow \text{CONCAVE DOWN}$$

[c] Find the co-ordinates of all points where the tangent line is vertical.

$\frac{dx}{dt} = 12t - 1 = 0 \longrightarrow t = \frac{1}{12} \quad \left(\left.\frac{dy}{dx}\right|_{t=\frac{1}{12}} \neq 0\right)$

$\left(-\frac{25}{24}, -\frac{5}{72}\right)$

[d] Find the co-ordinates of all points where the tangent line is horizontal.

$\frac{dy}{dt} = 4t - 1 = 0 \longrightarrow t = \frac{1}{4} \quad \left(\left.\frac{dy}{dx}\right|_{t=\frac{1}{4}} \neq 0\right)$

$\left(-\frac{7}{8}, -\frac{1}{8}\right)$